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DISCUSSIONS

THE SCORING OF WATER SUPPLIES

The writer has read with interest Mr. Wolman's several articles on "Index Numbers and Scoring of Water Supplies." The systematic collection of these data is certainly commendable. As to the best method of interpretation there may properly be some question. Certainly we should avail ourselves of the aid afforded by mathematics and statistics, and yet every effort should be made to maintain simplicity of expression, and this is indeed difficult. The averaging of results has the further defect of tending to obscure abnormalities, and one abnormal result may be more important than the average for a long period.

The index system proposed is based upon the B. coli test, and it is to the interpretation of this test that these remarks will be confined. Some years ago the writer became interested in trying to reason out what a percentage of positive tests really meant. The ensuing studies led from rather simple premises through rather involved mathematics to the following equation:

$$x = \log_e Q$$

where x represents the number of B. coli per sample, and Q the percentage of negative results in the series of tests made. The two fundamental assumptions made are that the number of B. coli in the water tested does not vary while the tests are being made, and that the number of tests made is sufficient to obtain representative results. This equation is the key for translating percentage results into actual numerical results which can be understood and dealt with in customary ways.

The mathematical deduction has been published elsewhere, but for non-mathematical readers it may be said that the only premise is the well-known "law of averages" as illustrated by the fact that a coin when tossed a large number of times will give "heads" as often as "tails." From this scientific fact, which most of us learned in early school-days, even though it was not in the curriculum, to the

above mentioned formula, the path is one of legitimate mathematics, which the non-mathematician must take on faith, with the further assurance that all mathematical processes are nothing more than short-cut methods of counting. After publishing, the writer learned that M. H. McCrady had published a method of interpreting B. coli results about a year before. A study of the McCrady formula showed that by eliminating certain assumptions as to size of sample, etc., it could be reduced to the above form. A month after the writer's article, a paper on the same subject was published by Greenwood and Yule in England, and again, their formulas can be reduced to the expression above. We therefore have three independent investigators leading to the same result. This covers all the recent work on interpretation of B. coli results, except that of W. F. Wells, which will be mentioned later. The Coli Index of Professor Phelps is admittedly only an approximation.

These matters are mentioned because the existence of several formulas has led to confusion of thought. Actually these are only variants of the formula given herein, into which the several authors have injected special constants, or which they have not reduced to final form. This formula must be the first step in any strictly logical interpretation of percentage results. Since, in the derivation it was assumed that the bacteria do not vary during the series of tests, and such a condition is seldom realized in fact, the direct application of this and all other proposed formulas to practical tests as usually conducted is very limited, and their indiscriminate use in unskilled hands will lead to erroneous results.

A careful study of the basic formula leads to results of utmost importance in carrying out and interpreting B. coli tests.

1. In order to avoid the indeterminate condition represented by 100 per cent. positive, each sample must contain not over 2 or 3 colon bacilli. This means that the results of fermentation tests are on a much lower scale of accuracy than the ordinary plate count; for example, the percentage positive in a water of constant composition determined by means of 200 fermentation tests will give the B. coli content about as accurately as a single plate will give the ordinary bacterial count.

2. Since fermentation tests are usually made in dilutions increasing as 1 to 10 to 100, it follows that only the percentage positive obtained from one of the three sets will lie in that portion of the curve representing the fundamental equation which gives determinate results.

Of the other two sets, one will usually be very close to 100 per cent positive and therefore practically indeterminate, and the other so close to zero as to be also practically indeterminate. Therefore, it is better to choose the most accurate of the three results and discard the other two. Any formula which obtains the average of all three dilutions, very greatly lowers the accuracy of the final result.

3. Since the fundamental formula assumes no variation in the concentration of coli while the tests are being made, it is necessary to inoculate a sufficient number of fermentation tubes at one time to obtain a percentage positive with some accuracy, in other words the "battery" method should be used. Any formula which claims to be able to derive rational results from the percentage of positive tests in a series extending over a considerable period of time, must make use of some occult science. This statement might be modified in the case of filtered, sterilized water, where in last analysis more reliance is placed on the purification processes than on the results of the tests.

As in all cases involving sampling, *B. coli* results are subject to error, due to the fact that insufficient tests are made to obtain representative results. Once the relationship between percentages and actual numbers has been established, the usual law of errors can be applied to these numbers, and the relative accuracy of results can be established. This the writer has treated so fully elsewhere that it is merely referred to here to point out another use of the fundamental equation.

Even though, as just said, we cannot draw definite conclusions from a series of tests extending over a period of time, still, it is possible to gain an insight into the workings of such a series of tests by assuming that the coli content varies in some definite way, and then computing the corresponding equation of translation. Several such results are shown graphically in figure 1. Curve I represents the fundamental equation with no variation in coli content, the others are plainly labeled, and also shown graphically by small sketches. Please note that it is not necessary for the coli content to pass consecutively through the values in the sketch curves, but may vary in a haphazard manner, provided that the values assume the proper curves when arranged in order of magnitude. It will be noticed that when the coli content follows the normal law, the curve approaches that for the fundamental equation, and since Mr. Wolman's research has shown that the *B. coli* in filtered water taken over a long period

approximate the normal law of variation, although somewhat asymmetrically, the legitimacy of using *B. coli* results for index purposes is fortified.

Just as the equation of translation is different for each assumed mode of variation of colon bacilli, so likewise does the probable error of sampling differ, constantly increasing as the variation becomes more pronounced, that is from Curve I to Curve VI. For example, suppose that in a series of 10 cc. tests, 50 per cent are positive. Then on the assumption that there was no variation during the tests, the *B. coli* would be 0.70 per sample and around this point a circle has

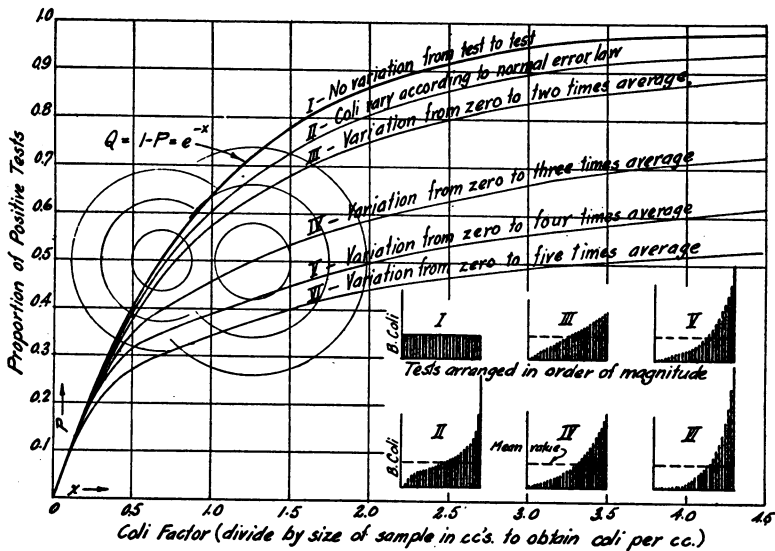


FIG. 1. CURVES OF EQUATION OF TRANSLATION FOR DIFFERENT MODES OF VARIATION OF *B. COLI*

been drawn representing the probable error. Similarly assuming a variation as in Curve IV, the result would be 1.3 per sample, and the probable error as shown. Since an error of three times the probable error is possible, the tests may indicate anything from 0.4 to 5.0 coli per sample, or from 4 to 50 per 100 cc. in this case.

It is not clear from Mr. Wells' several papers how he arrived at the use of the geometric mean as a *B. coli* index, but apparently it was in the nature of a "discovery" resulting from the study of numerous actual results, rather than by deductive methods. If in an

extended series of tests considered in sets, the proportion of negative tests in the several sets are represented by Q_1, Q_2, Q_3 , etc., and it is assumed that the corresponding numbers of *B. coli*, x_1, x_2, x_3 , etc., are obtained from the relation, $x_n = \log_e Q_n$, then the average x for the series

$$x = \sqrt[n]{x_1 + x_2 + x_3 + \dots + x_n}$$

but from the fundamental equation

$$x = \frac{\log Q_1 + \log Q_2 + \log Q_3 \dots}{n}$$

or

$$\log Q = \frac{\log (Q_1 \cdot Q_2 \cdot Q_3 \cdot \dots \cdot Q_n)}{n}$$

Therefore

$$Q = (Q_1 \cdot Q_2 \cdot Q_3 \cdot \dots \cdot Q_n)^{\frac{1}{n}}$$

That is to say, the average proportion of negative tests is the geometric mean of the proportions for the several sets. The same rule holds for the positive tests. There is nothing in this to take account of the time element, that is, the variation of *B. coli* content from test to test, which is a matter of chance, and cannot be accounted for in any formula, except that in the long run it approaches the normal distribution, as already pointed out.

Mr. Wells, however, applies the geometric mean directly to the number of *B. coli*, after it has been obtained by the method of Phelps or otherwise. The legitimacy of this seems to the writer doubtful. The geometric mean carries with it the idea of growth, such as the average population of a growing city over a period of time, or the average of a number of platings from a growing bacterial culture, etc. This is the way that the geometric mean and the time element are associated. But in the bacteriology of water, the bacteria found on one day are not the direct descendents of those found on previous days, the variation from day to day being due not to a regular growth, but to changing currents, run-off conditions, etc., and in the case of filter effluents to other equally accidental causes. Therefore it would seem more proper to use the arithmetic mean. Mr. Wells states that bacteriological results follow a logarithmic probability curve, which agrees with the writer's conclusions, first, because of the basic logarithmic equation given at the beginning of this dis-

cussion, and, second, in the case of tests extending over long periods the results are brought about by innumerable causative factors, each acting independently, and under such conditions a normal probability curve results, which is, of course, a logarithmic or exponential curve.

Perhaps, in this discussion, not enough emphasis has been laid upon isolated abnormal results, which the writer has considered elsewhere, including the "coli characteristic." Space forbids more than a mention of the fact that this is a matter of importance.

Mr. Wolman's studies are probably the most thorough made on the *B. coli* content of filtered water to date. His analytical methods and treatment of the results obtained are sound. It would seem to the writer that it might have been better to use ten tests instead of six per determination, and to use only one dilution. Inspection of the tabular data shows a very asymmetric frequency distribution, probably because the mean is so near the zero axis that there is no chance for a normal distribution without negative values. The reason that the logarithms of the *B. coli* contents seem to give a more normal distribution is because the small values then come further from zero relative to the large values. This is merely a geometrical condition of no special significance.

It is hoped that this discussion will assist somewhat in straightening out the confusion which seems to exist regarding the methods of interpreting *B. coli* results.

MILTON F. STEIN.¹

BACTERIAL UNIFORMITY INDEX

The close fit of the curves in Mr. Wolman's article on Index Numbers² is exceedingly valuable confirmation of the principle underlying the "Geometrical Mean" as a *B. coli* index. While these are probably the first published results showing the daily distribution of *B. coli*, the phenomenon was discovered in 1916 from an analysis of such data obtained in connection with oyster studies.

Professor Phelps has thrown light on the problem by distinguishing between the distribution of *B. coli* in space and its distribution in time. The former alone was discussed by McCrady (*Jour. Infect. Dis.*, 1915, 17, p. 183) in treating of fermentation tubes made from a single sample. The latter furnished

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² JOURNAL, November, 1920, pages 927-930.

the data for suggesting the "Geometrical Mean," which was based on a large number of samples taken at different times from single sources, as, for example, given points on a river. (Basis of the Geometrical Mean as a B. coli Index, *Science*, N. S., vol. xlix, no. 1269, p. 401, April 25, 1919.)

Associate Sanitary Engineer Pincus, analyzing a large number of similar results in connection with the Public Health Service investigation of Jamaica Bay, New Haven Harbor and Narragansett Bay, which, unfortunately, have not yet been published, also confirmed the conclusions.

If the phenomenon is correct, which Mr. Wolman's data also confirm, the "Geometrical Mean" follows as a logical B. coli index, and a very simple means of computing it is possible.

The reason the above simple methods give the "Geometrical Mean" arises from the fact that the ordinary bacteriological dilution scale is in reality a logarithmic scale, and the average dilution is the average logarithm or the logarithm of the "Geometrical Mean." A vast amount of published bacteriological data to be considered at some other time proves that bacteriological results follow a logarithmic probability curve, from which it follows that a median value closely corresponds to the geometrical mean. It is equally probable, therefore, that another sample would be greater or less than the geometrical mean. (The Geometric Mean as a B. Coli Index, *Science*, N. S., vol. xlvii, no. 1212, p. 48, January 11, 1918.)

The rule for obtaining the median value of the results which is, therefore, the geometric mean, called the B. coli index, becomes: Revert dilutions and apply Phelps' method. The direct application of Phelps' method to the results gives the arithmetical mean which, because of the nature of the distribution of the results, is always higher than the geometrical mean. The difference between the arithmetical and geometrical means varies directly with the uniformity of the results, and the ratio of the geometrical mean over the arithmetical mean gives a very valuable uniformity index. The greater the degree of uniformity of the results the more nearly do the arithmetical and geometrical means approach equality, and the more nearly does the ratio approach unity.

Applying this principle to Mr. Wolman's figures we obtain the results given in table 1. A study of the table will show that the uniformity index gives a correct expression of the nature of the variation. In the summary of three years' results which Mr. Wolman proposes as a fair case, the uniformity index is almost exactly 0.50. This states that if the geometric mean of the results is required to

pass the U. S. Treasury standard of 2 B. coli per 100 cc. of water, the arithmetic mean should not be more than double that figure or 4 B. coli per 100 cc. of water. This will be observed to be very approximately the case in the three-year averages given in the table.

The trail which Mr. Wolman is following, if persisted in, will give the reason for proposing "The Bacteriological Dilution Scale and

TABLE 1

Uniformity index of B. coli variation in samples of tap water during three years.
Geometrical Mean ÷ Arithmetical Mean = Uniformity Index

B. COLI PER 100 CC.		1917	1918	1919	SUMMARY 1917, 1918, 1919
Between	Assumed average				
From variation of <i>B. coli</i> content of all tap water samples					
0-2	1	132	146	178	441
3-5	4	63	32	5	94
6-10	8	39	13		47
11-20	15	19	5		20
21-30	25	7	0		5
31-40	35	1	2		2
41-50	45	1	0		0
51-100	75	3	2		3
100+	150	2	2		2
Total samples.....		267	202	183	614
Geometrical mean.....		2.78	1.65	1.04	1.71
Arithmetical mean.....		6.97	4.82	1.08	3.59
Ratio = Uniformity index.....		0.40	0.34	0.96	0.48
After eliminating abnormal results below line					
Total samples.....		262	196	183	609
Geometrical mean.....		2.59	1.55		1.65
Arithmetical mean.....		5.10	2.56		2.78
Ratio = Uniformity Index.....		0.51	0.67		0.59

the Dilution as a Bacteriological Unit" (*Jour. Am. Pub. Health Assn.*, September, 1919, pp. 664-667). With such a direct scale the average deviation of the dilutions positive would represent the uniformity index. Since, however, the mathematical interpretation does not correct for lack of precision of experimentation, a "Standard System of Bacteriological Dilutions" (*Jour. Am. Pub. Health Assn.*, December, 1919, pp. 956-959) has been proposed to make laboratory meth-

ods consistent with the resulting system of computation. "An Improved Fermentation Tube Battery" (*Jour. Am. Pub. Health Assn.*, December, 1918, pp. 904-905) will assist to simplify the bacteriological technique.

Conclusion.—"If dilutions are made in conformity with a standard dilution scale, and results computed in terms of dilutions, and interpreted directly according to their location on the scale, many of the present difficulties of bacteriological standards will disappear."

WILLIAM FIRTH WELLS.³

METERS FOR MULTIPLE-FAMILY HOUSES⁴

A private company differs somewhat from a municipal plant in that when new buildings go up in a municipal territory, building plans are generally submitted when an application is made to the water department for service and the department is immediately in possession of facts as to the size of the building, number of families and all information necessary to determine on the size of service to be installed. Such is not the case with a private company. In many cases the advice of our company is sought and almost invariably followed, but in more cases the application comes from the owner and the size is determined by either the plumber or architect.

It is difficult to lay down a hard and fast rule for the size of meters that will be workable in all cases, because there are so many things to be taken into account. The pressure condition is one. A pressure of 40 to 50 pounds will deliver through a $\frac{1}{2}$ -inch or $\frac{3}{8}$ -inch meter a certain quantity of water, but if that pressure is doubled the same rule does not apply. On the system of the Hackensack Water Company, where the elevations vary from 20 to over 500 feet, it is obvious that no fixed rule will fit all cases.

A municipal plant can go ahead and install a small service and set a small meter and if the service is not what it should be, or if any complaints are made with regard to its inadequate capacity, it can be changed and no complaints are made to the Public Utility Commission, for the reason that a municipal plant is outside of the jurisdiction of the Public Utility Commission. Whereas, if a private corporation installs a service which is too small to give a proper sup-

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⁴ Revision of oral discussion at Montreal convention, June 23, 1921.

ply it is likely to be followed by a complaint being filed with the Commission.

The practice the Hackensack Water Company has been following for the last several years has met with reasonable success. It has found that a $\frac{3}{4}$ -inch service installation from the street main to the meter and a $\frac{1}{2}$ -inch meter furnish satisfactory service to a two-family house. The company does not install services smaller than $\frac{3}{4}$ -inch inside diameter and a $\frac{1}{2}$ -inch meter will deliver as much water as a $\frac{3}{4}$ -inch service pipe will pass.

For buildings containing five to ten families, 1-inch services and 1-inch meters are installed and with a pressure of 50 to 60 pounds no complaints are received.

A large part of the Hudson County section of the system is developing rapidly into a tenement district, and buildings housing from twenty to one hundred families are numerous. In the larger of these four 1-inch services with a 1-inch meter on each service has been installed. The pressure is about 60 pounds. The buildings are five stories high and not a single complaint has been received because of lack of service.

Too little attention is often paid to the size of risers in large apartment houses. One might install a service and meter of suitable size, but unless the risers are ample enough to take proper care of the upper floors, or unless they are properly cross-connected, there is danger of complaints from the top floors. With connections such as were referred to, and with two cross connections on the fourth or fifth floors of a five-story building, the results have been so satisfactory that this practice is recommended unhesitatingly.

The installation of a suitable service and meter cannot overcome the effects due to improper interior plumbing. The use of black or wrought iron galvanized iron pipe is sure to result, in a few years at best, in decreased capacity on account of interior corrosion and scaling which finally lodges in elbows or elsewhere, partly choking the supply. When the piping approaches this condition we may expect "red water" trouble, especially so if the analysis shows the water to be low in alkalinity.

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